Interference in Thin Films: Interference in thin Parallel Film by Reflected Waves:

We know that the interference phenomena of light can be obtained by two ways – one by the way of division of wave front and another by the way of division of amplitude of the wave. Here we should mention that in case of interference in Young's double slit experiment, we adopt the method of division of wave front but in case of interference by thin film we basically use the method of division of amplitude to form the interference fringes.

When monochromatic light wave is allowed to incident to the surface of a thin transparent film having opposite parallel surfaces, say, parallel to each other (where as these two surfaces may be inclined to each other and in that case it is called wedge shaped thin film), then it will experiences successive reflection and refractions at both front and back surfaces of that thin film an finally both the reflected and transmitted part of the incoming



wave can give interference pattern seems to be appeared on the film surface.

Here as shown in figure, we consider two successive reflected waves, one is direct reflected wave from the point O_1 and another is indirect reflected wave from the point O_3 on the front surface of the film where both superimposes at the retina of human eye or at any screen point through eye lens or any converging lens at a certain path difference or phase difference and gives interference pattern on the retina where the eye will feel that to be appeared on the front surface of the film. This is interference by the thin film by the



reflected waves.

From the ray diagram we see that the path difference of that two reflected waves will be $\Delta = \mu(O_1O_2 + O_2O_3) - O_1P$ where μ is the refractive index of the material of the film

But also we have from the same figure if d be the thickness of that film then for angle of incidence i and first angle of refraction r we get $\mathbf{O}_1\mathbf{O}_2 = \mathbf{O}_2\mathbf{M} = \mathbf{M}\mathbf{O}_2$ and $\mathbf{M}\mathbf{O}_2 + \mathbf{O}_2\mathbf{Q} = \mathbf{M}\mathbf{Q} = 2\mathbf{d}\mathbf{Cosr}$.

Thus we finally get the path difference

$$\Delta = \mu(\mathbf{0}_1\mathbf{0}_2 + \mathbf{0}_2\mathbf{0}_3) - \mathbf{0}_1\mathbf{P} = \mu(\mathbf{M}\mathbf{0}_2 + \mathbf{0}_2\mathbf{Q} + \mathbf{Q}\mathbf{0}_3) - \mathbf{0}_1\mathbf{P} = \mu\mathbf{M}\mathbf{Q} + \mu\mathbf{Q}\mathbf{0}_3 - \mathbf{0}_1\mathbf{P}$$

But again we have from Snell's law of refraction

Sini =
$$\mu$$
Sinr $\Rightarrow \frac{O_1P}{O_1O_3} = \mu \frac{QO_3}{O_1O_3} \Rightarrow \mu QO_3 = O_1P$

 $\Delta = \mu MQ = 2\mu dCosr.$

Hence the path difference will then become

But here one thing is very important to note that for this interference by thin film which is made by denser transparent medium, the first direct reflected wave from the front surface of the film will suffer an additional phase change π by obeying Stoke's treatment.

Hence we have for this interference by thin film with parallel surfaces through the reflected waves; the phase difference is given by

$$\delta = \frac{2\pi}{\lambda}(\Delta) = \frac{2\pi}{\lambda}(2\mu dCosr) = 2n\pi + \pi = (2n+1)\pi \text{ for bright fringes}$$

And also $\delta = \frac{2\pi}{\lambda}(\Delta) = \frac{2\pi}{\lambda}(2\mu dCosr) = (2n+1)\pi + \pi = (2n+2)\pi = 2n'\pi$ for dark fringes

Interference in thin Parallel Film by Transmitted Waves:

Here we now consider two successive transmitted waves as shown in the figure when the



wave – I is directed transmitted wave and the wave – II is indirect one. Because of multiple reflections and refractions of the incoming wave at two parallel surfaces of the thin film we get these two successive waves which superimposes at a certain path difference or phase difference and then gives interference pattern seems to be appeared at the back surface of the film. This is interference in thin film by the transmitted wave.

From the ray diagram we see that the path difference of that two transmitted waves I and II will be $\Delta = \mu(0_20 + \mu)$

 $(00_3) - 0_2 P$ where μ is the refractive index of the material of the film. But also we have from the same figure if d be the thickness of that film then for angle of incidence i and first angle of refraction r we get

$$O_2O = OM = MO$$
 and $MO + OQ = MQ = 2dCosr$.

Thus we finally get the path difference

$$\Delta = \mu(O_2O + OO_3) - O_2P = \mu(MO + OQ + QO_3) - O_2P$$

= $\mu MQ + \mu QO_3 - O_2P$

But again we have from Snell's law of refraction

Sini =
$$\mu$$
Sinr $\Rightarrow \frac{O_2P}{O_2O_2} = \mu \frac{QO_3}{O_2O_2} \Rightarrow \mu QO_3 =$

Hence the path difference will then become

But here we see that since both the waves are transmitted waves, either direct or indirect transmitted, no wave is reflected from the denser medium which is also to be noted. Thus here Stokes treatment is not effective in this case of interference of thin film by the transmitted waves and normally we have for this interference by thin film with parallel surfaces through the transmitted waves; the phase difference is given by

 $\Delta = \mu MQ = 2\mu dCos$

$$\begin{split} \delta &= \frac{2\pi}{\lambda}(\Delta) = \frac{2\pi}{\lambda}(2\mu dCosr) = 2n\pi \text{ For bright fringes and also} \\ \delta &= \frac{2\pi}{\lambda}(\Delta) = \frac{2\pi}{\lambda}(2\mu dCosr) = (2n+1)\pi \text{ For dark fringes} \end{split}$$

Interference in Wedge Shaped Thin Films:

Here for this wedge shaped thin film, the two surfaces of the film are inclined with each other at a certain fixed angle and thus here also the interference phenomena can similarly be obtained by the both reflected and transmitted beam as before in case of thin film with parallel surfaces.



As we stated before that we so long considered fringes formed in the reflected and transmitted light when the plate or film is plane-parallel. But we will now consider the phenomenon, when the film is wedge-shaped, i.e., the surfaces enclosing the film are inclined at an angle α .

If we are now interested this interference in wedge shaped thin film by reflected wave we have from figure, PS_1 is the incident ray, reflected part goes along S_1O the transmitted part goes along S_1S_3 at an angle θ' , gets reflected along S_3S_2 , partly gets back in the incident medium along S_2O , such reflected rays unite producing constructive or destructive interference. S_2N_1 , S_2N_2 , S_2N_3 are all perpendiculars dropped from S_2 on to ray S_1S_3 , to ray S_1O and to the surface CD. The angles i, r, θ' and α are also shown in the figure.

Thus from figure $\beta + r = (90 - \theta') + \alpha$ and also $\beta + 2\theta' = 90^{\circ}$

So we have

$$90^{\circ} - 2\theta' + r = (90 - \theta') + \alpha \Rightarrow \theta' = (r - \alpha)$$

The path difference between the reflected rays

 $\Delta = \mu(S_1S_3 + S_3S_2) - S_1N_2 = \mu(S_1S_3 + S_3N_4) - S_1N_2$

$$= \mu S_1 N_4 - S_1 N_2 = \mu (S_1 N_1 + N_1 N_4) - S_1 N_2 = \mu S_1 N_1 + \mu N_1 N_4 - S_1 N_2.$$

Now from Snell's law, $\sin i = \mu \sin r \Rightarrow \frac{s_1 N_2}{s_1 s_2} = \mu \frac{s_1 N_1}{s_1 s_2} \Rightarrow S_1 N_2 = \mu S_1 N_1$

 $\Delta = \mu N_1 N_4 = \mu S_2 N_4 \cos \theta' = \mu \times 2d \cos (r - \alpha)$

where $d = S_2N_3$ represents thickness of the film at S_2 .Considering the path-difference introduced due to reflection from denser medium where the waves undergoes through additional phase difference π or path difference $\frac{\lambda}{2}$ by Stokes treatment, we can have the net path difference

$$\Delta_0 = \Delta \pm \frac{\lambda}{2} = 2\mu d\cos(r-\alpha) \pm \frac{\lambda}{2}$$

So for bright fringes, we get $\Delta_0 = 2\mu d \cos(r - \alpha) \pm \frac{\lambda}{2} = 2n \cdot \frac{\lambda}{2}$ and also for dark fringes $\Delta_0 = 2\mu d \cos(r - \alpha) \pm \frac{\lambda}{2} = (2n + 1) \frac{\lambda}{2}$.